

# A NOVEL APPROACH TO THE SYNTHESIS OF MIXED LUMPED AND DISTRIBUTED LOSSY NETWORKS

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**ABSTRACT:** In this paper, a general *lossy transformation technique*(LTT) is proposed for solving the problem of synthesis of *mixed lumped and distributed*(MLD) lossy networks. The elements, of which the MLD lossy networks are composed, can have arbitrary frequency-dependent losses. A computer-aided design example is presented to show the application of the new LTT to the design of a monolithic microwave integrated broadband FET amplifier with MLD lossy circuits as matching networks.

## I. INTRODUCTION

Recent developments in the theory and techniques of monolithic microwave integrated circuits(MMIC's) activated research and development efforts in both commercial and military communication systems and renewed interests in matching problems. However, conventional approaches to synthesizing and/or designing lossless matching networks become unsuited for MMIC's, for the losses of the matching elements fabricated on GaAs semi-insulating substrates are too large to be neglected. To solve this problem, a new technique (called lossy transformation technique) has been developed, which can be employed to effectively and exactly synthesize and/or design lumped and distributed lossy matching networks<sup>[1,2]</sup> with the elements, of which they are composed, having arbitrary frequency-dependent losses. But, no general theory has been found till now for handling a MLD lossy network, which has many outstanding merits in practical MMIC's, such as occupation of less chip area, convenience for integration, and so on. Only certain analytical methods for synthesizing MLD lossless networks were discussed by several authors<sup>[3-7]</sup> on the basis of multivariable positive real function theory and classical approximation methods. Since their approaches were quite complicated and only certain simple generator and load terminations could be treated, it was almost impossible to meet the needs of the more demanding and complicated matching problems. To author's knowledge, few papers dealt with the problem of synthesis of MLD lossy broadband matching networks, except for the paper written by Riederer<sup>[8]</sup>, in which only frequency-independent lossy elements were considered. Since the synthesis procedure introduced in that paper was based on the known driving-point impedance, then it has a little bit

This work was supported in part by Alexander von Humboldt Foundation and by National Natural Science Foundation of China.

difficulty in efficient design of practically applicable MLD lossy networks and has limited applicability to the modern broadband matching problems.

In this paper, a general lossy transformation technique, which will be simply called LTT thereafter, is proposed, with which the MLD lossy networks can be exactly and efficiently synthesized. Since arbitrary frequency-dependent losses can be included in the models of the elements, of which the MLD lossy networks are composed, then the theoretical performances of the MLD lossy networks are considerably close to measured ones. Finally, to exhibit the outstanding advantages of the new techniques presented in this paper, a monolithic microwave integrated broadband FET amplifier is designed with MLD lossy circuits as matching networks. We will see that the design procedure, which makes good use of real frequency technique<sup>[9,10]</sup>, is remarkably simplified and very suitable for engineering applications. It can be expected that the new technique introduced here may have even wider applications in the related fields of modern circuits and systems.

## II. GENERAL LOSSY TRANSFORMATION TECHNIQUE

Before presenting two new theorems and a corollary, which will indicate what kind of networks can be transformed to its corresponding lumped lossless reference network, a definition is given first for convenient descriptions thereafter.

**Definition:** Any lossy or lossless 2-port network, if it is passive, reciprocal and symmetric, will be denoted as a *building block*(BB).

**Theorem 1:** The transfer matrix of any BB can be expressed as

$$T = \begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix} = \frac{1}{\sqrt{1-z_1(s)/z_2(s)}} \begin{bmatrix} 1 & z_1(s) \\ 1/z_2(s) & -1 \end{bmatrix} \quad (1)$$

where  $A_{ij}$ , ( $i, j = 1, 2$ ) are the matrix elements and the functions of complex frequency variable  $s$ ;  $z_1$  and  $z_2$  are the short- and open-circuited input impedances of the BB.(The proof is omitted)

It can be seen from the theorem 1 that a BB may be totally determined by its short- and/or open-circuited input impedances. Moreover, the impedance matrix,  $Z$  of the BB can be calculated from the corresponding transfer matrix  $T$  of (1), via

$$Z = \frac{1}{A_{21}} \begin{bmatrix} A_{11} \Delta_T \\ A_{22} \end{bmatrix} = \begin{bmatrix} z_2 & z_2 \sqrt{1-z_1/z_2} \\ z_2 \sqrt{1-z_1/z_2} & z_2 \end{bmatrix} \quad (2)$$

where  $\Delta_T$  is the determinant of the  $T$ . Since  $z_1$  and  $z_2$  can always be expressed as

$$z_1 = Z_0, t z_1 \quad (3a)$$

$$z_2 = Z_0, t z_2 \quad (3b)$$

with  $Z_1$  and  $Z_2$  being the frequency-dependent parts of  $z_1$  and  $z_2$ , respectively, and  $Z_{0,t}$ , the common real positive multiplicative constant of  $z_1$  and  $z_2$ , then we can easily find in terms of the theorem<sup>[2]</sup> that all the impedance matrix elements satisfy the condition of the theorem, which is re-written here for convenience of the following explanation:

"If each element of a basic unit or of its equivalent circuit produces an individual impedance equal to the product of  $Z_2$  and a rational function of  $Z_1/Z_2$ , except for a possible branch point at  $Z_1/Z_2=1$ , where  $Z_1$  and  $Z_2$  are any physically realizable impedances..."

According to (2) and (3), it is found that a BB can be looked as a basic unit. Thus, the  $Z$  of the BB in the  $s$  domain, if it exists (i.e.,  $Z_1$  and  $Z_2$  are neither constant zero nor infinite number), can be transformed to the impedance matrix,  $\tilde{Z}$  of a corresponding "lumped" lossless reference unit element(UE), i.e.,

$$\tilde{Z} = \frac{Z}{Z_{0,t}} = \frac{Z_0, t}{\lambda} \begin{bmatrix} 1 & \sqrt{1-\lambda^2} \\ \sqrt{1-\lambda^2} & 1 \end{bmatrix} \quad (4)$$

with

$$\lambda = \sqrt{Z_1/Z_2} \quad (5)$$

being the function of  $Z_1$  and  $Z_2$  and being looked as a new complex frequency variable. Here,  $Z_{0,t}$  can be defined as the characteristic impedance of the reference UE in the  $\lambda$  domain.

The theorem 1 implies that the condition of the theorem<sup>[2]</sup> is always satisfied by a lossy or lossless 2-port network which is passive, reciprocal, and symmetric, and any BB can be transformed to a corresponding reference UE, and vice versa. Hence, a very useful corollary can be given as below:

**Corollary:** The condition of the lossy transformation theorem<sup>[2]</sup> is satisfied by all of the BBs and the transformation will always exist between the BBs and their corresponding "lumped" lossless reference UE.

In terms of this corollary, an important theorem can be achieved by extending the theorem<sup>[2]</sup> to even general case!

**Theorem 2:** any network  $N$ , if composed of the basic units which are the BBs of the same type and the BBs of this type short- and open-circuited at output port, can be transformed to a corresponding lumped lossless reference network  $\tilde{N}$  in the  $\lambda$  domain with the basic units in  $N$  corresponding to "lumped" lossless reference UEs, lossless reference inductors and capacitors, respectively.

To clearly exhibit the Theorem 2, consider a typical BB displayed in Fig.1(a), in which a length of transmission line with line length  $l$  is shunted by the same admittance  $Y(s)$  at both ends. The transfer matrix of the BB can be written as

$$\begin{bmatrix} 1-\mu(s)^2 & -1/2 \\ 1+Y(s)Z_0(s)\mu(s) & Z_0(s)\mu(s) \\ 2Y(s)+\{1+[Y(s)Z_0(s)]^2\}\mu(s)/Z_0(s) & 1+Y(s)Z_0(s)\mu(s) \end{bmatrix} \quad (6)$$

with

$$\mu(s) = \tanh[y(s)] \quad (7)$$

Since the line considered here has arbitrary frequency-dependent losses, then its characteristic impedance and propagation constant will be frequency-dependent, and can be given as<sup>[2]</sup>

$$Z_0 = Z_{0,t} \delta(s) \quad (8a)$$

$$Y = \rho_0 Y_0(s) \quad (8b)$$

$$\text{with } \delta(s) = \sqrt{(j\omega_n + 1/Q_{t1}) / (j\omega_n + 1/Q_{tc})} \quad (9a)$$

$$Y_0(s) = \sqrt{(j\omega_n + 1/Q_{t1}) (j\omega_n + 1/Q_{tc})} \quad (9b)$$

in which  $Z_{0,t}$  is the static characteristic impedance of the line when the losses of the line decrease to 0;  $\rho_0 = \omega_m / C_{vp}$ , the real positive multiplicative constant of the frequency-dependent part,  $Y_0$ , of the  $Y$ ;  $C_{vp}$ , the velocity of propagation on the line;  $Q_{t1}$  and  $Q_{tc}$ , the quality factors of the conductor and dielectric of the line at measured angular frequency  $\omega_m$ ; and  $\omega_n = \omega / \omega_m$ , the normalized angular frequency.

In general,  $Y(s)$ , the shunt admittance at either end of the line, may be any physically realizable positive real function. However, for the case now concerned in MMIC's, it may be the admittance of any inductor or capacitor with arbitrary frequency-dependent losses. For a lossy inductor or capacitor,  $Y(s)$  may be written as<sup>[1]</sup>

$$Y(s) = (1/L)Y_L(s) \quad (10a)$$

or

$$Y(s) = CY_C(s) \quad (10b)$$

with

$$Y_L = 1/[s + (i\omega\omega_m)^{1/2}/Q_1] \quad (11a)$$

and

$$Y_C = 1/[1/s + Q_C\omega_n^{1/2}/\omega_m + 1/(Q_C\omega)] \quad (11b)$$

respectively. Where  $L$  and  $C$  are the inductance and capacitance;  $Q_1$  and  $Q_C$  are the conductor losses of the inductor and the capacitor, and  $Q_C$ , the dielectric loss of the capacitor, respectively, all at measured angular frequency  $\omega_m$ . One can easily verify in accordance with the condition of the theorem<sup>[2]</sup> that the BB can be transformed to its corresponding reference UE as shown in Fig.1(a) with  $Z_{0,t}$  being the characteristic impedance of the UE. It should be indicated that  $z_1$  and  $z_2$ , the short- and open-circuited input impedances of the BB, as defined in the theorem 1, can be represented as

$$z_1 = \frac{Z_0(s)\mu(s)}{1+Y(s)Z_0(s)\mu(s)} = Z_{0,t}z_1 \quad (12a)$$

$$z_2 = \frac{Z_0(s)[1+Y(s)Z_0(s)\mu(s)]}{2Y(s)Z_0(s)+\{1+[Y(s)Z_0(s)]^2\}\mu(s)} = Z_{0,t}z_2. \quad (12b)$$

in which  $Z_1$  and  $Z_2$  are given as

$$Z_1 = \frac{\delta\mu}{1+\epsilon_1 Y_1 \delta\mu} \quad (13a)$$

$$Z_2 = \frac{1+\epsilon_1 Y_1 \delta\mu}{2\epsilon_1 Y_1 \delta\mu + \{1 + [\epsilon_1 Y_1 \delta\mu]^2\}\mu} \quad 1-L \text{ or } C \quad (13b)$$

with

$$\epsilon_L = Z_0, t/L \quad (14a)$$

$$\epsilon_C = Z_0, t/C \quad (14b)$$

being the design parameters and constrained to be the same for all the BBs of the same type, and  $Y_L$  or  $Y_C$  being the frequency-dependent parts of the  $Y(s)$ , as given in (11a) and (11b), respectively. Also, with the help of (13), the new frequency variable  $\lambda$  defined in (5) can be expressed in an explicitly form as

$$\lambda \sqrt{\delta [1 + (\mu^2 - 1) / (1 + \epsilon_1 Y_1 \delta\mu)^2]} \quad (15)$$

It differs from the expression of  $\mu(s)$  in (7), but will reduce to this expression if the BB is composed of only a length of lossy transmission line, i.e., the admittances shunted at both ends of the line in the BB are deleted. The attention, to which one should pay, is that when the BBs of the same type are employed as the basic units of the network  $N$ ,  $Z_{0,t}$ 's, which will be used as design parameters, may be different from each other for different lines in the BBs. But, in order to make the network  $N$  to satisfy the condition of the theorem<sup>[2]</sup>, the  $Z_1$  in all the BBs should be forced to be the same, so is the  $Z_2$ . To do so,  $\epsilon_L$ 's or  $\epsilon_C$ 's in the  $Z_1$  and  $Z_2$  should be identical. Thus, the network  $N$  composed of all the BBs can be transformed a corresponding lumped lossless network  $\tilde{N}$  with each basic unit in  $N$  as shown in Fig.1(a) corresponding to a reference UE shown in Fig.1(a); on the other hand, the short- and open-circuited input impedances of the BBs can also be written in the similar forms as (3a) and (3b), respectively,

$$Z_{sh} = Z_0, sh Z_1 \quad (16a)$$

$$Z_{op} = Z_0, op Z_2 \quad (16b)$$

here,  $Z_{0,sh}$  and  $Z_{0,op}$  are the real positive multiplicative constants of  $Z_1$  and  $Z_2$ , respectively. They will also be used as the design parameters as  $Z_{0,t}$  and may be different from each other for different short-circuited BBs or for different open-circuited BBs. Of course, the BBs mentioned here should be of the same type. Thus, the short- and open-circuited BBs can also be employed as the basic units of the network  $N$ . Since the condition of the theorem<sup>[2]</sup> can be satisfied by the network  $N$  which are constructed by these basic units, then the network  $N$  can as well be transformed to a corresponding lumped lossless reference network  $\tilde{N}$  in the  $\lambda$  domain with the short- and open-circuited BBs shown in Fig.1(b) and (c) corresponding to their lossless reference inductors and capacitors, respectively, as shown in Fig.1(b) and (c), as long as the  $\epsilon_L$ 's or  $\epsilon_C$ 's in the short- and open-circuited BBs are forced to be the same as those in the BBs. For other commonly used BBs, their topologies and expressions for  $z_1$  and  $z_2$  as defined in (1) are omitted due to the limited pages. It will be seen in next Section how the LTT described above is

applied to achieving the scattering matrix of the lossy network  $N$  from that of a supposed lumped lossless reference network  $\tilde{N}$ .

### III. DESIGN EXAMPLE

In this section, an example is presented to show the application of the LTT to the synthesis and/or design of a single stage monolithic microwave integrated broadband FET amplifier with MLD lossy circuits as input and output matching networks. We first supply an interactive computer program which is developed in terms of the technique described above with both generator and load data, (i.e.,  $Z_G, Z_L = 50\Omega$ ), and measured FET scattering parameters, which are identical to those given in [11] over an octave passband 4-8GHz. Then the BB constructed by a line shunted with the same capacitors at both ends, and its short- and open-circuited structures, are employed as the basic units of our MLD input and output matching networks. Since the losses of the line and capacitors in the BB are considered in the synthesis of the MLD lossy matching networks, then  $Q_{t1}$  and  $Q_{tc}$ , which represent the conductor and dielectric losses of the lines as given in (9) are specified to be 80 and 120, respectively, at measured frequency  $f_m = 8\text{GHz}$ . Therefore, after the line length  $l$ , which is employed as a variable in our optimization, is initialized as  $\lambda_m/16$ , where  $\lambda_m$  is the wavelength at 1.5 times the high-frequency limit of the passband, the  $\delta$  and  $\mu$  in (14) will be determined in terms of (7), (8b), and (9), at certain real frequency  $\omega$ . As for the conductor and dielectric losses of the capacitors, which are expressed by the quality factors,  $Q_c$  and  $Q_d$ , are specified to be 100 and 50, respectively. Hence,  $Y_C$  in (14) can also be calculated by means of (11b) within the passband. In consideration of the  $\lambda$  which should be still valuable in lossless case (i.e., when  $Q_{t1}$  and  $Q_{tc}$  become infinite, the  $\lambda$  should be imaginary), the  $\epsilon_C$  of (14b) employed also as a variable in the optimization, is chosen as  $10\text{pF}$  in our example. Since  $\delta$ ,  $\mu$ ,  $Y_C$ , and  $\epsilon_C$  in (13) have been achieved,  $Z_1$  and  $Z_2$  which will be looked as the functions of  $\omega$  and optimizing variables  $l$  and  $\epsilon_C$ , can then be computed. For the sake of practical simplicity, the scattering parameters,  $\tilde{e}_{11, v}(\lambda)$  ( $v=1,2$ ) of supposed lumped lossless reference network are assumed to have the following forms:

$$\tilde{e}_{11, v} = h(\lambda)/g(\lambda) \quad (17a)$$

$$\tilde{e}_{12, v} = \tilde{e}_{21, v} = (+/-) \lambda^k (1 - \lambda^2)^{m/2} / g(\lambda) \quad (17b)$$

$$\tilde{e}_{22, v} = (-1)^{k+1} h(-\lambda) / g(\lambda) \quad (17c)$$

where

$$h(\lambda) = h_1 + h_2 \lambda + \dots + h_{n+1} \lambda^n$$

is an arbitrary polynomial with real coefficients  $h_i$  ( $i=1, 2, \dots, n+1$ ) being optimizing variables and initialized as  $h_1 = +/-1$ , and

$$g(\lambda) = g_1 + g_2 \lambda + \dots + g_{n+1} \lambda^n$$

is a Hurwitz polynomial. If the numbers of high-pass elements, UEs, and total matching elements, which correspond to  $k(0)$ ,  $m(0)$ , and  $n(k+m)$ , respectively, are specified, the  $g(\lambda)$  can then

be uniquely determined from the  $h(\lambda)$  in terms of the lossless property of the  $e_{1,1,v}$ . The subscript  $v$ , which is equal to 1 or 2, stands for the input or output matching network. In accordance with the LTT demonstrated in Section II, it is evident that the lossy transformation formula of (11) in [1], which has been successfully applied to obtaining scattering matrices of lumped and distributed lossy matching networks<sup>[1,2]</sup>, can also be employed to achieve the scattering matrix of MLD lossy matching network. Therefore, the scattering matrix,  $E(s)$  of the MLD lossy matching network can be achieved from the corresponding scattering matrix,  $E(\lambda)$ , of a supposed lumped lossless reference matching network, via<sup>[1]</sup>

$$E = \sqrt{Z_1 Z_2 (I + \tilde{E}) - (I - \tilde{E})} / \sqrt{Z_1 Z_2 (I + \tilde{E}) + (I - \tilde{E})} \quad (18)$$

where  $I$  is the identity matrix;  $Z_1$  and  $Z_2$  are specified by (14) and  $\tilde{E}$  composed of the  $e_{1,1,v}$ , ( $1, j=1,2$ ) of (17). So far, the TPG of the lossy matched FET amplifier can be computed by means of the definition and expressions given in [1] which are applicable to the synthesis and/or design of both lossy and lossless matched FET amplifiers, and the goal flat gain level  $T_0$ , which is specified to be 7dB by calculating the stable conjugate gain of the FET within the passband, will be approached by an optimization routine. After optimization, the topologies of the supposed lumped lossless input and output reference matching networks can be obtained by synthesizing the unit normalized reflection factors  $e_{1,1,1}$  and  $e_{1,1,2}$ . Because the basic units of the MLD

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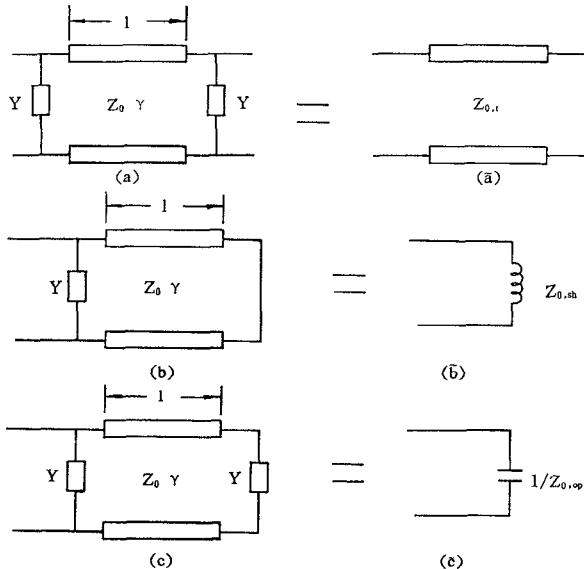


Fig.1 (a) and (b) A BB constructed by a length of line shunted by the same admittance at both ends and its corresponding "lumped" lossless reference UE; (b) and (c) The BB of the same type short-circuited at output port and its corresponding lossless reference inductor; (c) and (c) The BB of the same type open-circuited at output port and its corresponding lossless reference capacitor

matching networks in the  $s$  domain should be able to correspond to their respective reference UEs, inductors, and capacitors in the  $\lambda$  domain, as described in Section II, the MLD lossy matching networks shown in Fig.2 can be achieved through substituting the BBs and short- and open-circuited BBs for the reference UEs, inductors, and capacitors, respectively. Thus, the design of a monolithic microwave integrated broadband FET amplifier with practically realizable MLD lossy matching networks is finally accomplished. From the Table, which lists the values of all the elements in Fig.2, an advantage of the MLD network can be found that the line lengths of the distributed elements are much shorter than those as usually defined by [1], and this advantage has been verified by designing lots of similar MLD networks. Therefore, the chip area on which the amplifier will be developed can be greatly reduced. On the other hand, since the intolerant losses of the matching elements in MMIC's, which may be in certain cases larger than those we have assumed and can not be omitted in the synthesis and/or design of monolithic microwave integrated circuits, are directly considered in our network design, then the design procedure is simplified and the performance of the amplifier shown in Fig.3 will be considerably close to measured one of the practically realized amplifier. If without considering the losses in the elements, the lossless gain performance is also remarkably better than the initial gain response given in [11] due to outstanding merit of the real frequency technique we have utilized<sup>[9, 10]</sup>.

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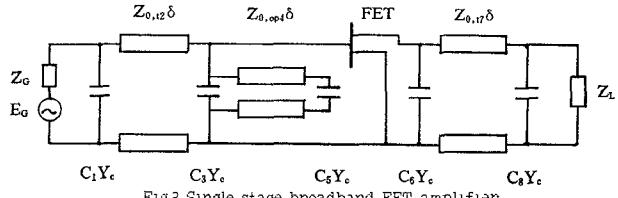


Fig.2 Single stage broadband FET amplifier

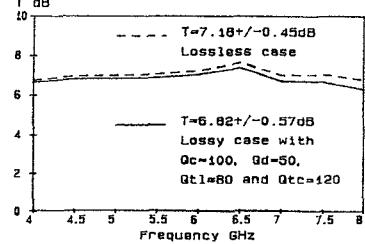


Fig.3 The frequency response of the amplifier

Table Amplifier parameters

Fig. 6(a)		Fig. 6(b)		Fig. 6(a)		Fig. 6(b)	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
$C_1$	0.3561pF	$C_1$	0.1769pF	$t_{c,1}$	1.106	$t_{c,1}$	1.106
$Z_{0,12}$	61.781Ω	$Z_{0,12}$	124.405Ω	$C_4$	0.0504pF	$C_4$	0.0504pF
$C_2$	0.3513pF	$C_2$	0.3513pF	$Z_{0,12}$	75.788Ω	$Z_{0,12}$	75.788Ω
$Z_{0,op1}$	62.628Ω	$Z_{0,op1}$	126.133Ω	$C_3$	0.0501pF	$C_3$	0.0504pF
$C_4$	0.3513pF	$C_4$	0.1774pF	$l_1$	1.065mm	$l_1$	1.065mm
$l_1$	1.762mm	$l_1$	1.762mm	$t_{c,2}$	0.192	$t_{c,2}$	0.192